Quantum Communication and Satellite Networks: Extending Entanglement Beyond the Horizon

Abstract

Quantum communication via satellites represents the most promising route toward a global quantum network. By transmitting entangled photons through free space rather than optical fibre, satellites overcome terrestrial loss limits and enable secure quantum key distribution (QKD) across continents. This essay explores the mathematical and physical foundations of satellite-based quantum communication, focusing on quantum entanglement, decoherence, and the geometry of photon propagation in curved spacetime. It situates satellite quantum links within the broader framework of quantum information theory and relativistic quantum mechanics.

1. Introduction

Classical communication relies on electromagnetic waves governed by Maxwell's equations. Quantum communication, by contrast, encodes information in quantum states - typically single photons -whose behaviour is described by the Schrödinger equation and the postulates of quantum mechanics. The challenge is to preserve quantum coherence over long distances. Fibre-optic channels suffer exponential attenuation, with transmission probability $T \propto e^{-\alpha L}$, where $\alpha$is the absorption coefficient and $L$the distance. Satellites bypass most of the atmosphere, reducing loss and enabling global entanglement distribution.

2. Quantum States and Entanglement Distribution

A pair of entangled photons can be represented as:

$$| \Psi^- \rangle = \frac{1}{\sqrt{2}} (| 0\rangle_A | 1\rangle_B - | 1\rangle_A | 0\rangle_B)$$

where $| 0\rangle$and $| 1\rangle$denote orthogonal polarisation states. When one photon is transmitted to a ground station and the other remains on a satellite, the joint state remains entangled provided decoherence is minimal. The fidelity $F = | \langle \Psi_{\text{ideal}} | \Psi_{\text{actual}} \rangle |^2$quantifies how well the entanglement survives transmission.

Atmospheric turbulence introduces phase noise $\phi(t)$, modelled as a stochastic process with variance $\sigma_\phi^2 \propto L^{5/6}$. Adaptive optics and temporal filtering are used to minimise this decoherence.

3. Quantum Key Distribution (QKD) via Satellites

The BB84 protocol is the canonical QKD scheme. Two bases are used:

$$\mathcal{B}_1 = \{|0\rangle, |1\rangle\}, \mathcal{B}_2 = \{|+\rangle, |-\rangle\}$$

where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

In satellite QKD, the probability of successful key generation depends on the link efficiency η, given by:

$$\eta = \frac{A_r}{4\pi R^2} \cdot T_{\text{atm}} \cdot \eta_d$$

where A_r is the receiver aperture area, R the satellite-to-ground distance, T_{atm} the atmospheric transmission coefficient, and η_d the detector efficiency. For low-Earth-orbit satellites ($R \approx 500$ km), typical values yield $\eta \sim 10^{-6}$, sufficient for secure key exchange given modern photon sources.

4. Relativistic Effects and Quantum Frames

Quantum communication across orbital distances requires accounting for relativistic time dilation and gravitational redshift. The frequency shift between satellite and ground is:

$$\frac{\Delta f}{f} = \frac{GM}{c^2}\left(\frac{1}{r_g} - \frac{1}{r_s}\right)$$

where G is the gravitational constant, M Earth's mass, c the speed of light, and r_g, r_s the radii of ground and satellite orbits. This shift affects photon energy $E = hf$ and must be corrected to maintain phase coherence in entangled states. The correction is implemented via local unitary transformations $U = e^{-i\Delta\phi\sigma_z/2}$.

5. Decoherence and Quantum Channel Capacity

The quantum channel capacity C_q is limited by noise and loss. For a depolarising channel with error probability p:

$$C_q = 1 - H_2(p)$$

where $H_2(p) = -p\log_2 p - (1-p)\log_2(1-p)$ is the binary entropy. Satellite channels exhibit lower p than fibre links, enabling higher C_q over global distances. Experimental results from the *Micius* satellite demonstrated entanglement distribution over 1,200 km with fidelity $F > 0.8$, confirming theoretical predictions.

6. Toward a Global Quantum Internet

The integration of quantum satellites with terrestrial fibre networks forms the backbone of a future quantum internet. Each satellite acts as a **quantum repeater**, extending entanglement through successive teleportation steps:

$$| \psi\rangle_{AB} \otimes | \phi\rangle_{BC} \xrightarrow{\text{Bell measurement}} | \psi\rangle_{AC}$$

This process relies on Bell-state measurements and classical communication, constrained by the no-cloning theorem. The global network thus becomes a lattice of entangled nodes - a physical realisation of distributed quantum computation.

7. Conclusion

Satellite-based quantum communication transforms the geometry of information. It merges the relativistic framework of orbital mechanics with the probabilistic structure of quantum theory. Mathematically, it unites Hilbert spaces, group symmetries, and field propagation under a single architecture of coherence. Physically, it extends entanglement beyond the horizon, turning the planet itself into a quantum system.

The future quantum internet will not merely transmit data; it will transmit correlation, the fundamental currency of quantum mechanics. In that sense, every photon sent from orbit is not just a signal - it is a mathematical statement about the connectedness of the universe.